Endogenising demand for information in road transport

Emmerink, R.; Verhoef, E.T.; Nijkamp, P.; Rietveld, P.

published in
The Annals of Regional Science
1996

DOI (link to publisher)
10.1007/BF01581973

Ann Reg Sci (1996) 30:201–222

The Annals of
Regional Science

Endogenising demand for information in road transport

Richard H.M. Emmerink*, Erik T. Verhoef*,, Peter Nijkamp, Piet Rietveld**

Department of Spatial Economics, Free University, De Boelelaan 1105, 1081 HV Amsterdam, The Netherlands (e-mail: remmerink@econ.vu.nl, Tel.: +31/20/4446098, Fax: +31/20/4446005)

Received: June 1995 / Accepted: January 1996

Abstract. In this paper, the impact of endogenous information provision to drivers in road transport is investigated. A static economic equilibrium model is used, which allows potential road users to buy information on the prevailing (stochastic) traffic situation. It takes for granted that an indiviual will try to acquire proper information when the private benefits of doing so exceed the private costs. By using an information model for road users, the interesting result is found that the provision of endogenous information leads to a strict Pareto improvement. Furthermore, the model shows that – depending on the price of information – it can be efficiency improving to subsidise or tax the motorist information to the user. Finally, there is a relationship between fine congestion pricing and subsidising motorist information. It turns out that the social welfare maximising subsidy under first-best congestion pricing is equal to zero. However, subsidising information may be an attractive policy instrument when a flat congestion pricing scheme is preferred.

1. Introduction

Traffic congestion is one of the most pressing transportation problems, particular in urban areas. The negative impacts of congestion are not strictly confined to the transportation sector, but are affecting the economy as a whole. Various ways to resolve (part of) the congestion problem in the transport sector have been addressed in the literature.[1] In the present paper we will focus on one of these solu-

[1] There is a large body of literature on congestion in transport networks. For useful references see Arnott et al. (1993), and Johansson and Mattsson (1995).

* Affiliated to the Tinbergen Institute

** Participates in the VSB-fonds sponsored research project "Transport and Environment" tion strategies: the provision of information to improve the efficiency of road usage. This tool is becoming increasingly more important as more traditional ways of resolving the congestion problem (such as expansion of the existing road network) are viewed as infeasible, due to the negative social and environmental consequences (Boyce 1988).

Research assessing the effects of these new information technologies has usually focused on (1) the technical feasibility (witness the many technical projects within the European Community DRIVE I and II programme); (2) the impact on driver behaviour using a wide variety of methodologies (Bonsall (1992); Emmerink et al. (1996); Kobayashi (1994); Lotan and Koutsopoulos (1993); Yang et al. (1993)); and (3) the impact on the efficiency of road usage (see, for example, Emmerink et al. (1995a), and Mahmassani and Jayakrishnan (1991)).

In the present paper, the latter issue using a static economic equilibrium model, in which individual decisions are interdependent due to a congestion externality, is investigated. To model the large and often unpredictable random fluctuations in levels of congestion, the link travel cost functions are assumed to be random variables. Two types of actors are being considered: informed and uninformed ones. Informed actors are assumed to have perfect knowledge on the realisation of these random variables, and are therefore basing their trip-making decision on actual costs. Uninformed actors, in contrast, do not have this information, and hence base their behaviour on expected costs. However, in contrast to previous work (Emmerink et al. 1994a, 1995b), we assume that there are costs associated with information provision which reflect, for instance, the costs of the necessary information technology equipment. In this manner, the choice of being informed is modelled endogenously, whereas in previous papers this process was modelled as an exogenous input. Hence, an actor in the present model does not only decide on whether or not to use the transport network, but also decides upon whether to buy information on the traffic situation. Clearly, an actor will buy the information only if the private benefits of being informed at least exceed the private costs of doing so.

This model will also be used to examine the efficiency improving properties of two types of government regulation. First, we will consider the possible implications of subsidising the costs of information for social welfare. The idea is that owing to the external benefits generated by the information to uninformed actors, it may be attractive for the government (or an infrastructure authority) to subsidise information (Emmerink et al. 1994b) and we will analyse under which conditions it is socially desirable for the government to do so. Next, we will investigate the link between fine congestion pricing and endogenous provision of information. Fine congestion pricing will yield its first-best characteristic, only if the users of the system are perfectly aware of the prevailing fine congestion toll. Without perfect information on actual levels of congestion and tolls, users would base their behaviour on expected costs rather than on actual costs. Therefore, it seems logical to consider the efficiency of first-best congestion tolling in combination with the endogenous provision of perfect information. With respect to the second point, we will elaborate the analysis of El Sanhouri (1994) and Verhoef et al. (1994), who assumed that information is available for free, both for all actors and for the government.

The structure of the paper is as follows. In the next section we will first present a version of a model in which information is modelled exogenously. In Sect. 3 we will turn to the case where the choice of being informed is endogenous, while the above mentioned regulatory issues are treated in Sect. 4. Finally, Sect. 5 contains the conclusions.

2. Exogenous demand for information

First we will present the model in which the choice of being informed is an exogenous input. The polar cases of (1) all drivers and (2) no driver being informed will be presented in order to be able to identify the factors underlying the (derived) demand for information. We will first assume that all drivers are perfectly informed on the stochastic conditions in a one-link network. The demand function for road usage is given by D, and stochasticity is introduced by means of uncertainty in terms of the link travel cost function. There are two link travel cost functions giving the private costs of road use: one representing so-called recurrent congestion (a link travel cost function C^0 for state 0), and one representing non-recurrent or stochastic congestion (a link travel cost function C^1 for state 1). It is assumed that state 1 implies both relatively high link travel costs and relatively high congestion costs. Hence, the relationship:

$$C^0(N) \leq C^1(N) \quad \text{and} \quad \frac{\partial C^0(N)}{\partial N} \leq \frac{\partial C^1(N)}{\partial N} \tag{1}$$

holds for all feasible levels of road usage N. It is further assumed that state 0 takes place with probability $1-p$, while state 1 occurs with probability p.

When drivers are perfectly informed on the prevailing state, they will adjust their travel choice decision according to this knowledge. This situation might, for instance, be visualised by a motorist information device that provides perfect information to individuals equipped with the technology. Given the static equilibrium nature of the model, the type of information given can best be interpreted as pre-trip information.

The model is presented in expressions (2) and (3), and is fully in line with generally accepted economic theory stating that an informed individual will use the network if private benefits exceed private costs. Moreover, this conforms to Wardrop's first principle (see Wardrop (1952)) for the user equilibrium, as both may be characterized by individual maximizing behaviour. In expressions (2) and (3), a subscript *ex* (exogenous) refers to the fact that information is exogenously provided to all actors, and superscripts 0 and 1 refer to the two possible states (0: low link travel costs; 1: high link travel costs).

$$D(N^0_{ex}) \leq C^0(N^0_{ex}) \ , \quad N^0_{ex} \geq 0 \quad \text{and} \quad N^0_{ex} \cdot (D(N^0_{ex}) - C^0(N^0_{ex})) = 0 \tag{2}$$

$$D(N^1_{ex}) \leq C^1(N^1_{ex}) \ , \quad N^1_{ex} \geq 0 \quad \text{and} \quad N^1_{ex} \cdot (D(N^1_{ex}) - C^1(N^1_{ex})) = 0 \tag{3}$$

For the present discussion it is important to focus on the *internal* and *external* benefits and costs of information provision. Internal information benefits are benefits owing to better decision-making by the informed driver himself, while external information benefits to an arbitrary driver arise from the fact that *other* road users are being informed on the traffic situation.

For obtaining the internal and external benefits and costs of information provision, we have to derive the model in which no information is available. In this model it is assumed that all (potential) road users base their behaviour on *expected* link travel costs rather than on *prevailing* costs. Hence, uninformed drivers may be characterized as frequent commuters who are familiar with the average traffic conditions. This model is presented in expression (4).

$$D(N_n) \leq (1-p)\cdot C^0(N_n) + p\cdot C^1(N_n)\ , \quad N_n \geq 0 \quad \text{and}$$
$$N_n\cdot(D(N_n) - ((1-p)\cdot C^0(N_n) + p\cdot C^1(N_n))) = 0 \tag{4}$$

Here, subscript n (no information) refers to the model without information. For a more rigorous analysis of the above type of models we refer to Emmerink et al. (1994a). In their study it was, inter alia, shown that information provision to an exogenously determined fraction of potential road users leads to a strict Pareto improvement, see Fig. 1.

The internal and external benefits and costs of information provision are depicted in Figs. 1 and 2. In these figures, a link travel cost symbol C without superscript denotes the expected link travel costs ($C = (1-p)C^0 + pC^1$), and should be evaluated at the relevant equilibrium level of trip demand, for example, $C_{ex} = (1-p)C^0(N^0_{ex}) + pC^1(N^1_{ex})$.

Figure 1 demonstrates the property that information provision will lead to a strict Pareto improvement. Information benefits for all drivers are non-negative, and positive for those on the left-hand side of N^0_{ex}. Next, in Fig. 2 the information benefits (and costs) are separated out over the two possible states of nature. First, consider the left-hand panel of Fig. 2. As discussed in Emmerink et al. (1994a), in state 0, drivers on the left-hand side of N_n incur additional *external* congestion costs due to the information provided (when information is provided there are *more* drivers using the network in state 0), while drivers between N_n

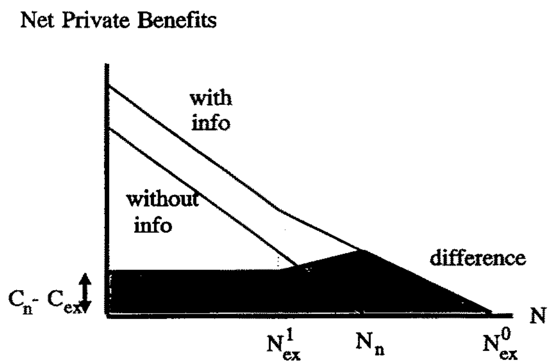

Fig. 1. Expected net private benefits from information

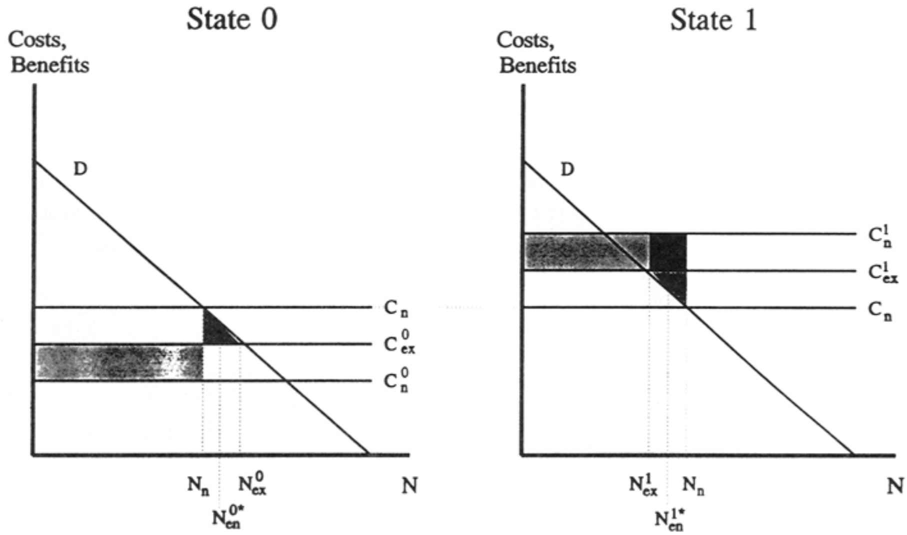

Fig. 2. Welfare effects of information

and N^0_{ex} incur *internal* decision-making benefits (when information is provided these drivers do *not* use the network). Alternatively, when state 1 is prevailing (the right-hand panel), drivers to the left-hand side of N^1_{ex} benefit from an *external* decrease in congestion costs, while those between N^1_{ex} and N_n benefit both from an *external* decrease in congestion costs and *internal* decision-making benefits. Although the first of the effects mentioned above is clearly negative, the other effects are all positive, and it can be shown that the total impact of information provision on overall welfare is non-negative, as illustrated in Fig. 1 and formally proved in Emmerink et al. (1994a).

3. Endogenous demand for information

Thus far, we have assumed that the decision of being informed is exogenous, i.e. is not determined within the model. In this section we will present a model in which the choice of being informed is endogenous, and dependent on the *internal* benefits that an *informed* individual derives from the information.

3.1. The model

In the previous section we have seen that there are two kinds of information benefits, *internal* and *external*. In the model to be presented hereafter, we will assume that an individual decides to be informed when the *internal* benefits of information exceed the private costs of being informed. It is rational to consider the internal benefits only, because for an arbitrary individual the *external* benefits are (by definition) independent of whether or not that particular individual is himself informed. These external benefits (and costs) are caused by the fact that *other* road users are informed on the prevailing traffic situation. This observation implies that only drivers between N^1_{ex} and N^0_{ex} are potentially interested in being informed, since these are the ones that may incur internal benefits from the information. Drivers at the left-hand side of N^1_{ex} are not interested, since they will use the road network regardless the prevailing state, whereas drivers to the right-hand side of N^0_{ex} are not interested since they will use the network in neither state.

Now consider driver N^{0*}_{en} in the left-hand panel of Fig. 2, where the subscript *en* (endogenous) refers to the model in which the demand for information is endogenous. If for this driver the internal benefits of being supplied with information exceed the costs, then the same holds for all drivers between N_n and N^{0*}_{en}, since for these drivers internal benefits are larger than those of driver N^{0*}_{en}. A similar reasoning holds for driver N^{1*}_{en} in the right-hand panel of Fig. 2. If it is beneficial for this particular driver to be informed, then this is the case for all the drivers between N^{1*}_{en} and N_n. Therefore, if we denote the marginally informed driver (the driver who is indifferent between being informed or not) on the left-hand side of N_n with N^1_{en}, and the marginally informed driver on the right-hand side of N_n with N^0_{en}, then in equilibrium N^0_{en}-N^1_{en} drivers are informed.

The resulting network situation in the model with endogenous demand can now be characterised as follows. First, the informed drivers (those between N^1_{en} and N^0_{en}) will only use the network when state 0 is prevailing, and will thus fully benefit from their *internal* decision-making benefits. Second, drivers on the left-hand side of N^1_{en} will not buy the motorist information equipment, and will always use the transport network. This can easily be seen by noting that they would do the same in case no information was available for equipped drivers. Finally, drivers to the right-hand side of N^0_{en} will not buy the motorist information equipment and will use the network in neither state.

In order to determine N^0_{en} and N^1_{en}, we need of course the price of information. Due to the static equilibrium nature of the model, the price of information π to be considered below is short of any time dimension. In other words, whereas one would intuitively think of π as an individual investment, the internal benefits of which were to be reaped during a subsequent (large) number of travel decisions, such reasoning is not in the spirit of static equilibrium analysis. Hence, for the translation of the present model into more practical terms, one should either interpret the diagrams in Figs. 1 and 2 as daily cost and benefit curves and π as the daily equivalent of some purchase price Π (where π reflects daily interest and depreciation), or one could consider π as the real purchase price and take D and C to be some discounted measures of the future stream of benefits and costs of road usage.

The mathematical formulation of the model is presented below. Since, by definition, the marginally informed driver is indifferent between being informed or not, the following two relationships should hold:

$$(1-p)\cdot(D(N^0_{en})-C^0(N^0_{en}))=\pi \tag{5}$$

$$p\cdot(C^1(N^1_{en})-D(N^1_{en}))=\pi \tag{6}$$

where π denotes the private costs of information. The term in the large parentheses in expression (5) gives the internal decision-making benefits for the marginally informed driver N^0_{en} in state 0. Hence, the *expected* internal decision-making benefits are given by multiplying this term with the probability that state 0 occurs. The left-hand side of expression (6) gives the expected internal decision-making benefits for a marginally informed driver N^1_{en}. Obviously, the expected internal decision-making benefits of the marginally informed driver should equal the private costs of information π.

Also, notice that for both N^0_{en} and N^1_{en}, expressions (5) and (6) guarantee that the expected net private benefits of the marginally *uninformed* driver and marginally *informed* driver are the same, and equal to 0 for N^0_{en}, and equal to $(1-p)(D(N^1_{en})-D(N^0_{en}))$ for N^1_{en}. Therefore, both the marginally informed driver N^0_{en} and N^1_{en} are indifferent in buying the information.

To derive the properties of the model discussed above, we will assume that both the demand and cost curves are linear functions over the relevant ranges considered, i.e. $D(N) = d-aN$ and $C^j(N) = k^j+b^jN$ for $j = 0, 1$.[2] Then solving for N^0_{en} and N^1_{en} in expressions (5) and (6) yields:

$$N^0_{en} = \frac{d-k^0}{a+b^0} - \frac{\pi}{(1-p)\cdot(a+b^0)} = N^0_{ex} - \frac{\pi}{(1-p)\cdot(a+b^0)} \tag{7}$$

$$N^1_{en} = \frac{d-k^1}{a+b^1} + \frac{\pi}{p\cdot(a+b^1)} = N^1_{ex} + \frac{\pi}{p\cdot(a+b^1)} \tag{8}$$

where π is in the interval $[0, \pi_{\max}]$ to ensure that N^1_{en} is smaller than N^0_{en}.[3] The first term on the right-hand side of expression (7) [(8)] gives the number of informed drivers using the network in state 0 (state 1), in case the information were provided for free. The second term on the right-hand side of these expressions captures the effect of the price of information.

The proposition below presents some of the properties of the model with endogenised demand for information compared to the model in which no information is available.

Proposition. *In a one-link network with endogenised demand for information, and assuming linear demand (D) and cost (C^0, C^1) functions, $C^0(N) \leq C^1(N)$, and $dC^0(N)/dN \leq dC^1(N)/dN$, the following relationships hold:*

- *expected road usage is higher with information than without;*
- *expected link travel costs are smaller with information than without;*
- *expected welfare is higher with information than without.*

Proof. See Appendix A.1.

[2] It is interesting to note that Arnott, De Palma and Lindsey (1992) showed that these cost functions include as a special case the reduced form of Vickrey's dynamic bottleneck model.
[3] See Appendix A.1 for a derivation of $\pi_{\max}$.

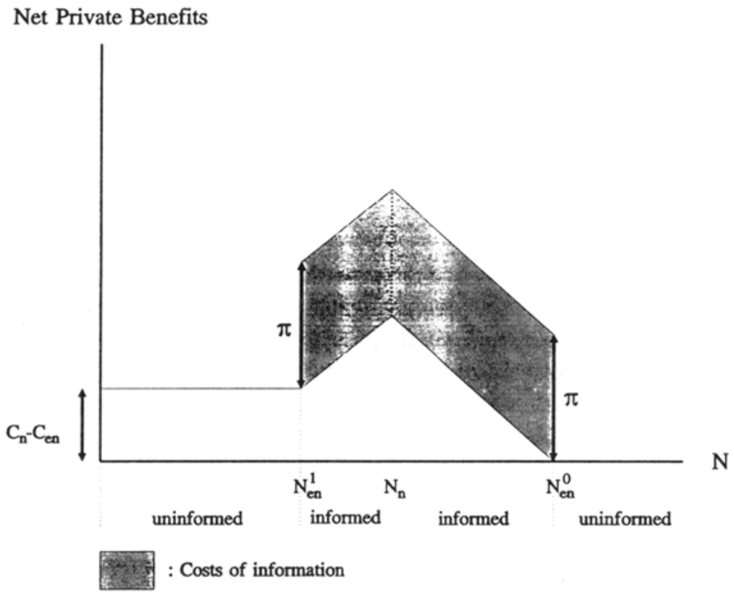

Fig. 3. Difference in expected net private benefits of the model with endogenous demand for information and the model in which no information is available

The net private benefits of *endogenous* provision of information compared to the situation in which no information is available are depicted in Fig. 3. Drivers on the left-hand side of N^1_{en} benefit from an *external* decrease in congestion costs of the size C_n minus C_{en}, where $C_m (m = n, en)$ denotes the expected link travel costs in model m. Drivers between N^1_{en} and N_n benefit in addition from *internal* decision-making benefits owing to the purchased information. Finally, drivers between N_n and N^0_{en} benefit from *internal* decision-making benefits solely.

Using the equilibrium levels of road usage as given by expressions (7) and (8) when the private costs of being provided with information are equal to π, we can derive the demand curve for information. Given a price of information of π, N^0_{en} minus N^1_{en} drivers would like to be supplied with information. Therefore, the relationship between the number of informed drivers and the private costs of information is given by:[4]

$$D^{-1}(\pi) = N^0_{ex} - N^1_{ex} - \pi \cdot \left(\frac{1}{(1-p) \cdot (a+b^0)} + \frac{1}{p \cdot (a+b^1)} \right) . \tag{9}$$

The term in large parentheses multiplied by the costs of being equipped with a motorist information device π denotes the number of drivers for whom the costs

[4] Strictly speaking, the notation D^{-1} may be a bit confusing as (9) gives the actual demand curve, and not its inverse. However, as we used D before to denote inverse demand (in line with common practice), we have to live with this slight inconvenience.

of information exceed the internal benefits of being informed. The term N^0_{ex} minus N^1_{ex} reflects the number of drivers who are interested in being informed when the information is provided for free.

3.2. Welfare effects

Next, we will explore the welfare properties of the model with endogenous demand for information. In the Proposition in Sect. 3.1 we have already seen that the *availability* of information will lead to an increase in social welfare, defined by total system benefits minus total system costs. The manner in which this is accomplished is graphically shown in Fig. 4.

Figure 4 depicts the relationship between the private costs of information, the number of informed drivers, the expected link travel costs, and expected welfare. For example, a price of information equal to π leads to N_π informed drivers, expected link travel costs will then be equal to C_π, and expected welfare equal to W_π. The upper panel of Fig. 4 shows the linear demand function for information as given by expression (9), where $\pi_{\max}$ is the price for which demand is equal

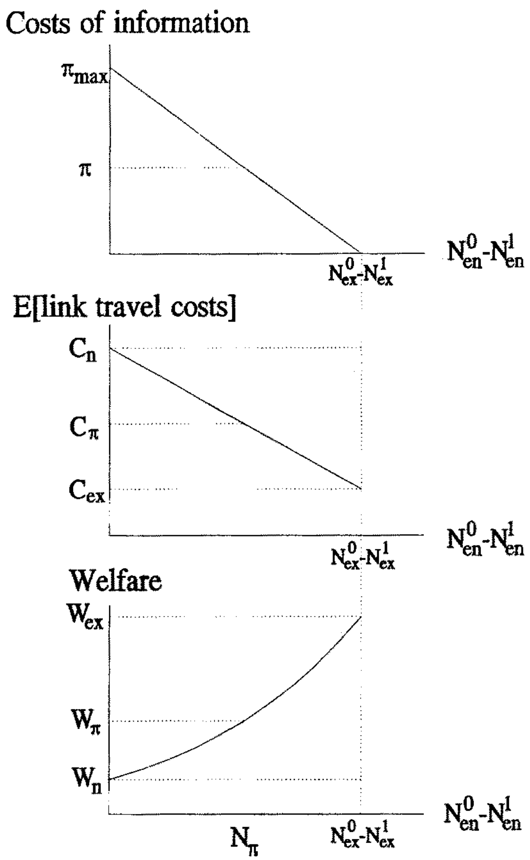

Fig. 4. Welfare effects of endogenous information

to zero (see Appendix A.1 for a derivation of $\pi_{\max}$). Then, the middle panel of Fig. 4 gives the expected link travel costs ($C_{en} = (1-p)C^0(N^0_{en})+pC^1(N^1_{en})$) as a function of the number of informed drivers, which is of course a function of the price of information. With linear D and C^j ($j = 0, 1$) curves, it can be shown that this relationship is linear as well. Clearly, expected link travel costs run from a minimum of C_{ex} (where information is free, so that every one is willing to be informed) to a maximum of C_n (where information is so expensive that no one is willing to be informed).

Finally, the lower panel of Fig. 4 depicts expected welfare as a function of the number of informed drivers, which is a function of the price of information. Under the assumptions made it can be shown that this is an increasing convex quadratic function, running from a minimum when no one is informed (W_n), to a maximum when every one is informed (W_{ex}) (see Appendix A.2 for a formal proof). The convexity of the expected welfare curve implies that as the number of informed drivers increases, then so does social welfare, and even at an increasing rate. Three effects are playing a role here. First, as the number of informed drivers increases, welfare for the "newly" informed drivers increases. If this were not the case, then they would not have bought the information in the first place. Second, owing to more informed drivers in the network, the expected link travel costs decrease, leading to additional benefits for the uninformed drivers. Finally, in the model an increase in the number of informed drivers can only be realised when the private costs of information π decrease, thereby increasing total welfare.

4. Regulatory issues

In the previous sections we have seen that information is not only beneficial to the (endogenously) informed drivers, but in addition to the (endogenously) uninformed ones, because the expected link travel costs in the network will decrease (see Fig. 3). From economic theory it is well known that such kinds of external effects may distort the efficiency of the market system in economic processes. In these circumstances, the market outcome without government intervention will generally not coincide with the allocation that maximises social welfare. Hence, the existence of external effects to uninformed drivers renders the issue of subsidising motorist information systems relevant. In the following sections we will use the previously presented model to analyse this in greater detail. First, in Sect. 4.1 we will derive the optimal level of the subsidy on motorist information equipment; next (Sect. 4.2), we will extend the scope of the analysis towards a situation in which a motorist information system exists in combination with fine congestion tolling. Finally, we will compare these two types of policy instruments from an efficiency point of view in Sect. 4.3.

4.1. Subsidising information without tolling

The optimal subsidy s for the motorist information equipment, given the price of information π, can be found by maximising expected welfare subject to individual maximising behaviour:

$$\max_s (1-p)\cdot\left(\int_0^{N_{en}^0} D(x)dx - C^0(N_{en}^0)\cdot N_{en}^0\right)$$
$$+p\cdot\left(\int_0^{N_{en}^1} D(x)dx - C^1(N_{en}^1)\cdot N_{en}^1\right) - \pi\cdot(N_{en}^0 - N_{en}^1)$$

subject to

$$\begin{aligned}(1-p)\cdot(D(N_{en}^0)-C^0(N_{en}^0)) &= \pi - s\\ p\cdot(C^1(N_{en}^1)-D(N_{en}^1)) &= \pi - s\end{aligned} \tag{10}$$

The two restrictions in (10) ensure that information is allocated in an individually rational manner. Furthermore, due to the subsidy s, the costs of information as paid for by the users of the information system have decreased from π to $\pi-s$. Also, note that the objective function is not explicitly dependent on the subsidy s, since from a welfare economic point of view, the redistributive impact of the subsidy is not relevant for the policy's efficiency. The optimal subsidy s then follows from solving the following Lagrangian:

$$\begin{aligned}\mathscr{L} = {} & (1-p)\cdot\left(\int_0^{N_{en}^0} D(x)dx - C^0(N_{en}^0)\cdot N_{en}^0\right) + p\cdot\left(\int_0^{N_{en}^1} D(x)dx - C^1(N_{en}^1)\cdot N_{en}^1\right)\\ & -\pi\cdot(N_{en}^0 - N_{en}^1)\\ & +\lambda^0\cdot((1-p)\cdot(D(N_{en}^0)-C^0(N_{en}^0)) - \pi + s)\\ & +\lambda^1\cdot(p\cdot(C^1(N_{en}^1)-D(N_{en}^1)) - \pi + s)\end{aligned} \tag{11}$$

The five necessary first-order conditions are given by:

$$\begin{aligned}\frac{\partial\mathscr{L}}{\partial N_{en}^0} &= (1-p)\cdot(D(N_{en}^0) - C^0(N_{en}^0) - C^{0\prime}(N_{en}^0)\cdot N_{en}^0)\\ &\qquad -\pi + \lambda^0\cdot(1-p)\cdot(D'(N_{en}^0) - C^{0\prime}(N_{en}^0)) = 0\\ \frac{\partial\mathscr{L}}{\partial N_{en}^1} &= p\cdot(D(N_{en}^1) - C^1(N_{en}^1) - C^{1\prime}(N_{en}^1)\cdot N_{en}^1)\\ &\qquad +\pi + \lambda^1\cdot p\cdot(C^{1\prime}(N_{en}^1) - D'(N_{en}^1)) = 0\\ \frac{\partial\mathscr{L}}{\partial s} &= \lambda^0 + \lambda^1 = 0\\ \frac{\partial\mathscr{L}}{\partial\lambda^0} &= (1-p)\cdot(D(N_{en}^0) - C^0(N_{en}^0)) - \pi + s = 0\\ \frac{\partial\mathscr{L}}{\partial\lambda^1} &= p\cdot(C^1(N_{en}^1) - D(N_{en}^1)) - \pi + s = 0\end{aligned} \tag{12}$$

This set of first-order conditions can be solved to yield the following (implicit) expression for the optimal subsidy:

$$s=\frac{\gamma^1\cdot w^0-\gamma^0\cdot w^1}{w^0+w^1} \tag{13}$$

where

$$\begin{aligned}
\gamma^0 &= (1-p)\cdot C^{0\prime}(N^0_{en})\cdot N^0_{en}\\
\gamma^1 &= p\cdot C^{1\prime}(N^1_{en})\cdot N^1_{en}\\
w^0 &= (1-p)\cdot(D'(N^0_{en})-C^{0\prime}(N^0_{en}))\\
w^1 &= p\cdot(D'(N^1_{en})-C^{1\prime}(N^1_{en}))
\end{aligned} \tag{14}$$

For linear demand and cost curves the optimal subsidy can be given explicitly by:

$$s=\frac{N^1_{en}(\pi)\cdot\left(\dfrac{b^1}{a+b^1}\right)-N^0_{en}(\pi)\cdot\left(\dfrac{b^0}{a+b^0}\right)}{\dfrac{a+2b^0}{(1-p)\cdot(a+b^0)^2}+\dfrac{a+2b^1}{p\cdot(a+b^1)^2}} \tag{15}$$

where $N^j_{en}(\pi)$ denotes the equilibrium levels of road usage with a price of information equal to π and no subsidy ($j=0,\ 1$).

After substituting $N^j_{en}(\pi)(j=0,\ 1$; see Eqs. (7) and (8)) into expression (15), it follows that in the model with linear demand and cost curves the optimal subsidy is an increasing linear function of the price of information π. Hence, the higher the price of information, the higher the optimal regulator's subsidy. This linear relationship, however, may also imply that the optimal value of the subsidy may be negative when the price of the information is relatively low. This then leads to the interesting result that it might be welfare increasing to *tax* rather than to *subsidise* information! At first sight, this may seem a counter-intuitive result, since we found in previous sections that information provision leads to positive external effects to uninformed drivers as well. However, it should be realised that information induces *more* travel in the network, while in order to achieve system optimal behaviour in case of congestion, road usage should be *reduced*. Apparently, when the optimal subsidy is negative, it is more efficient for the government to use the motorist information system as an instrument to price for congestion rather than to stimulate the use of information.

To conclude, the optimal subsidy captures two effects. On the one hand, a positive value of the subsidy will induce more drivers to be informed, and hence will generate a more efficient use of the network. On the other hand, a negative subsidy (a tax) is a kind of (second-best) instrument to price for congestion, and hence to reduce overall levels of road usage. The optimal subsidy is a compromise between these two effects, and turns out to be more in favour of the latter at a relatively low price of information, and more in favour of the former at relatively high prices.

Using the parameter values from Emmerink et al. (1994a), a typical situation is depicted in Fig. 5.[5] With these parameter values, $\pi_{\max}$ (the maximum possible price of information under which demand for information is still positive, see Appendix A.1) is approximately equal to 3.88.

The linear increasing relationship between the price of information π and the optimal subsidy s is given in the upper panel of Fig. 5. The bottom panel depicts the mapping between the price of information π and expected welfare. Here, we have compared expected welfare with and without government intervention to optimally subsidise motorist information systems. Clearly, expected welfare with the imposition of the optimal (welfare maximising) subsidy is at least as high as without, and both are (of course) identical when the optimal subsidy is equal to zero. In the bottom panel of Fig. 5 it is important to note that the difference in expected welfare under the two types of regulation is very small in our model. The difference is significant at only relatively low and high prices of information π. A conclusion regarding this point is that (for a wide range of parameter values) our model indicates that subsidising information as a tool to achieve a more effi-

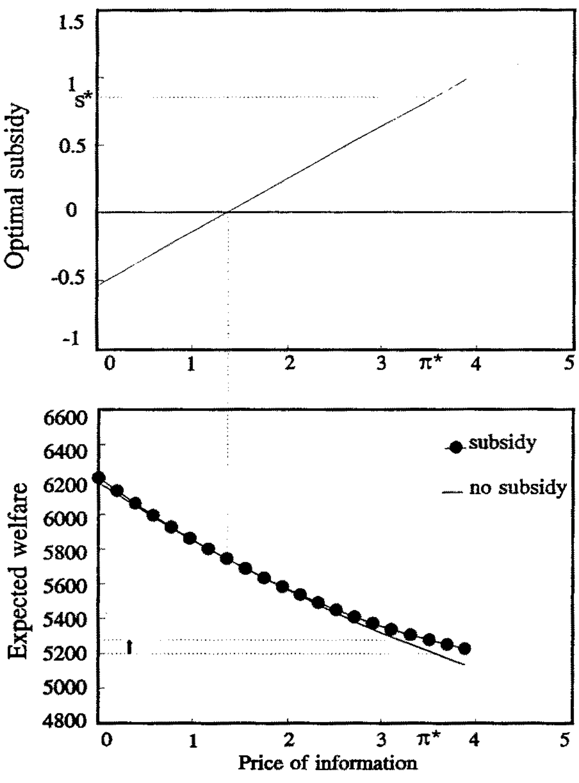

Fig. 5. The relationship between the price of information, subsidising information, and expected welfare

[5] The following parameter values were used to generate Fig. 5: $d = 50$, $a = 0.015$, $k^0 = k^1 = 20$, $b^0 = 0.015$, $b^1 = 0.04$, and $p = 0.25$.

cient allocation of road usage may not always be very effective. In Sect. 4.3 we will return to this issue by comparing other types of government regulation as well. Before doing so, we will first introduce fine (fluctuating) congestion-pricing in combination with endogenous information in the next section.

4.2. Combining fine congestion tolling and subsidising motorist information

The first-best solution in congested transport networks – the solution that is optimal from a social welfare point of view – can be achieved by implementing a fine (i.e. dependent on the level of congestion) congestion pricing scheme. By doing so, marginal private costs will coincide with marginal social costs, so that individually optimal behaviour is also in line with overall optimality. In order to obtain these socially desired behavioural responses, it is necessary for all potential road users to be perfectly aware of the prevailing level of the fine tolls. Hence, some kind of information system is needed to ensure that this first-best policy will indeed achieve what it is intended to. Or in other words, proper fine tolling cannot be implemented without the simultaneous implementation of some kind of information system.

In the present section, we will derive the optimal fine congestion tolls (f^0: in state 0; f^1: in state 1) and the optimal motorist information subsidy s simultaneously, by assuming that cost price of information is equal to π. The present analysis is based on earlier work by Verhoef et al. (1994), and it takes properly into account the costs of providing information.

In the model, the government will determine the fine fees f^0 and f^1, and the optimal level of the subsidy s in order to maximise expected social welfare. Hence, the regulator faces the following maximisation problem:

$$\max_{f^0,f^1,s}\ (1-p)\cdot\left(\int_0^{N_{en}^0} D(x)\,dx - C^0(N_{en}^0)\cdot N_{en}^0\right) + p\cdot\left(\int_0^{N_{en}^1} D(x)\,dx - C^1(N_{en}^1)\cdot N_{en}^1\right) - \pi\cdot(N_{en}^0 - N_{en}^1) \tag{16}$$

subject to

$$(1-p)\cdot(D(N_{en}^0)-C^0(N_{en}^0)-f^0) = \pi - s$$

$$p\cdot(C^1(N_{en}^1)+f^1-D(N_{en}^1)) = \pi - s$$

The two restrictions in (16) ensure that individual behaviour is rational, i.e. no individual can increase expected net private benefits by changing behaviour, and hence the solution is an equilibrium. Clearly, these restrictions follow directly from expressions (5) and (6) after taking proper account of the introduction of fine congestion fees f^0 and f^1, and the subsidy for the motorist information system s.

The optimal fine fees and subsidy for information can be found by solving for the following Lagrangian:

$$\begin{aligned}\mathscr{L} = & (1-p)\cdot\left(\int_0^{N_{en}^0} D(x)\,dx - C^0(N_{en}^0)\cdot N_{en}^0\right)\\ & +p\cdot\left(\int_0^{N_{en}^1} D(x)\,dx - C^1(N_{en}^1)\cdot N_{en}^1\right)\\ & -\pi\cdot(N_{en}^0 - N_{en}^1)\\ & +\lambda^0\cdot((1-p)\cdot(D(N_{en}^0) - C^0(N_{en}^0) - f^0) - \pi + s)\\ & +\lambda^1\cdot(p\cdot(C^1(N_{en}^1) + f^1 - D(N_{en}^1)) - \pi + s)\end{aligned} \tag{17}$$

The seven necessary first-order conditions are given by:

$$\begin{aligned}\frac{\partial\mathscr{L}}{\partial N_{en}^0} &= (1-p)\cdot(D(N_{en}^0) - C^0(N_{en}^0) - C^{0\prime}(N_{en}^0)\cdot N_{en}^0)\\ &\qquad -\pi + \lambda^0\cdot(1-p)\cdot(D'(N_{en}^0) - C^{0\prime}(N_{en}^0)) = 0\\ \frac{\partial\mathscr{L}}{\partial N_{en}^1} &= p\cdot(D(N_{en}^1) - C^1(N_{en}^1) - C^{1\prime}(N_{en}^1)\cdot N_{en}^1)\\ &\qquad +\pi + \lambda^1\cdot p\cdot(C^{1\prime}(N_{en}^1) - D'(N_{en}^1)) = 0\\ \frac{\partial\mathscr{L}}{\partial s} &= \lambda^0 + \lambda^1 = 0\\ \frac{\partial\mathscr{L}}{\partial f^0} &= -\lambda^0\cdot(1-p) = 0\\ \frac{\partial\mathscr{L}}{\partial f^1} &= \lambda^1\cdot p = 0\\ \frac{\partial\mathscr{L}}{\partial \lambda^0} &= (1-p)\cdot(D(N_{en}^0) - C^0(N_{en}^0) - f^0) - \pi + s = 0\\ \frac{\partial\mathscr{L}}{\partial \lambda^1} &= p\cdot(C^1(N_{en}^1) + f^1 - D(N_{en}^1)) - \pi + s = 0\end{aligned} \tag{18}$$

The following expressions for the optimal fine fees and subsidy can be derived:

$$\begin{aligned}f^0 - \frac{s}{1-p} &= C^{0\prime}(N_{en}^0)\cdot N_{en}^0\\ f^1 + \frac{s}{p} &= C^{1\prime}(N_{en}^1)\cdot N_{en}^1\end{aligned} \tag{19}$$

There are two equations to solve for three variables, implying that we can fix one variable and then solve for the other two. For example, fixing the subsidy at 0, leads to:

$$f^0 = C^{0\prime}(N^0_{en})\cdot N^0_{en}$$
$$f^1 = C^{1\prime}(N^1_{en})\cdot N^1_{en} \tag{20}$$

The expressions in (20) show that once fine congestion tolling equal to marginal external congestion cost takes place, there is no need for subsidising motorist information systems. In this case, individuals face the optimal incentives for the decision of whether to be informed. Furthermore, it can be seen that these optimal fine fees are given by the traditional expression reflecting the external costs of congestion, i.e., the external costs that the marginal driver imposes on the other drivers.

However, the expressions in (19) also show that there are more (an infinite number of) ways to reach the socially optimal level of road usage. For example, when information is subsidised, the corresponding optimal fee in state 0 is somewhat larger, and the fee in state 1 is smaller. This can intuitively be explained by noting that when information is subsidised, then more drivers are willing to be informed. This in turn will imply that more people will be inclined to use the network when state 0 prevails, and hence, the fine fee in state 0 will be set at a higher level. Similarly, less people will be inclined to use the network in state 1, so that the fine fee in state 1 will be smaller.

An important implication of the above reasoning is that in the model presented above, flat congestion pricing (which means that $f^0 = f^1$, so that the fine is independent of the level of congestion in a particular state) in combination with the optimal subsidy for motorist information equipment (see expression (21) for the optimal flat fee and subsidy) will lead to the social welfare maximising solution. Under the additional, and rather weak, assumption that the free flow travel costs are identical in both states – $C^0(0)$ is equal to $C^1(0)$ –, it can easily be proven that the optimal subsidy s corresponding to the flat fee scheme is always positive, since under this assumption $C^{1\prime}(N^1_{en})N^1_{en}$ minus $C^{0\prime}(N^0_{en})N^0_{en}$ is always greater than zero. Therefore, the subsidy will never be a tax. Moreover, the expression of the flat fee in (21) shows that it can be interpreted as the expected external congestion costs.

$$f^0 = f^1 = (1-p)\cdot C^{0\prime}(N^0_{en})\cdot N^0_{en} + p\cdot C^{1\prime}(N^1_{en})\cdot N^1_{en}$$
$$s \;= p\cdot(1-p)\cdot(C^{1\prime}(N^1_{en})\cdot N^1_{en} - C^{0\prime}(N^0_{en})\cdot N^0_{en}) \tag{21}$$

The reason that the combination of a flat fee and a subsidy is as efficient as fine tolling is of course closely related to the fact that the subsidy is in equilibrium only enjoyed by actors using the network in state 0. It therefore enables the regulator to discriminate perfectly between actors for whom it is efficient to use the network only in state 0, and those for whom it is efficient to use it in both states.

Furthermore, it can be shown that all the available welfare maximising strategies, as given by the expressions in (19), lead to the same expected net private benefits for the actors (the uninformed drivers, the informed drivers, and the regulator) in the system. This follows from the fact that the expected net private benefits for all actors in the system are independent of the optimal subsidy s.

Hence, the subsidy on information cannot be used by the regulator to influence the equity distribution among actors under a first-best policy.

The results obtained above cannot easily be generalised to more complex networks and more states of nature. For instance, in a one-link three-state transport network, the optimal flat fee and optimal subsidy on information will not yield the system optimum, since in that particular situation there are only *two* instruments (the flat fee and the subsidy) to deal with *three* different traffic situations. However, in Verhoef et al. (1994) in a two-link network, it was shown that the combination of information provision and flat tolling is a rather robust instrument, and worked out to be almost as good as the theoretically first-best option of fine tolling. Moreover, given the public and political opposition to fine congestion-pricing, flat pricing in combination with subsidising information may be an attractive policy option.[6]

In the next section, we will compare the welfare effects of the first-best regulatory pricing and information scheme discussed above with the second-best policy as presented in Sect. 4.1.

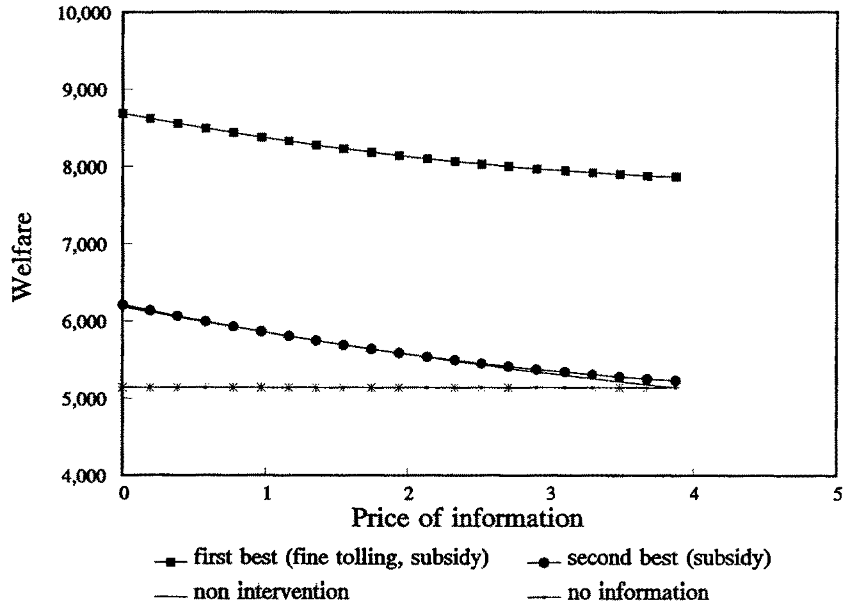

Fig. 6. Relationship between the price of information and welfare

[6] See for arguments pro and contra congestion-pricing Johansson and Mattsson (1995). Furthermore, for empirical evidence that suggests public aversion against pricing in a fluctuating manner, see the arguments in Frey and Pommerehne (1993). These authors found that a price rise due to a non-recurrent shock (such as a sudden excess in demand) is considered unfair by 80 percent of the respondents in their survey.

4.3. *Efficiency of regulation*

The welfare effects of four different policies will be compared. As previously, well fare is defined as total system benefits minus total system costs. The four policies to be compared are:

- first-best: the regulator sets optimal fine tolls; the price of information is π;
- second best: there are no tolls, but the regulator sets optimal subsidy on information; the price of information is π;
- non intervention: the regulator does not intervene, but information is available at a price π;
- no information: the regulator does not intervene, and information is not available.

In Fig. 6, the impacts of these four policy options on welfare are depicted as a function of the price of information, using the same parameter values as in Sect. 4.1.

Clearly, the curve labelled *no information* is independent of the price of information, and is therefore a horizontal line. The *non intervention* curve crosses the *no information* one at $\pi_{\max}$ (≈ 3.88), since there are no informed actors at this price of information. Next, notice that, as in Fig. 5, the impact of subsidising information (without fine tolling) is negligible. Finally, it is important to note that welfare can be increased quite substantially under *first-best* transport policy (optimal fine tolls and no subsidy, or optimal flat toll and optimal subsidy). As argued in Verhoef et al. (1994), information provision without some kind of congestion pricing will in most cases not direct the traffic flows towards a level that is close to the system optimum.

5. Concluding comments

In this paper, we have analysed the impact of the endogenous provision of information to potential road users in transport networks. To do so, a static economic equilibrium model was used, which allowed potential road users to buy information on the prevailing stochastic traffic situation. An individual driver will acquire information, only if the private benefits of being informed exceed the private costs. Then, uninformed potential road users base their trip-making behaviour on the expected costs, while informed ones use the actual costs.

By comparing this model with the one in which no information is available, it was proven that the endogenous provision of information leads to a strict Pareto improvement. The size of this increase in social welfare depends strongly on the private costs of information. When the private costs of information decrease, then more actors are willing to be informed, which in turn decreases expected link travel costs and increases social welfare.

Next, given a certain price of information, the optimal subsidy was derived, i.e. the subsidy that maximises social welfare. It was found that the optimal subsidy increases with the price of information. However, the optimal subsidy can be negative when the information is very cheap. Then, it is from a government point of view more efficient to use the motorist information system as an instrument to price for congestion rather than to stimulate a more efficient use of the network. In both situations, however, it was shown that the potential welfare improvement of subsidising (or taxing) information is relatively small.

The analysis of the relation between fine congestion pricing and subsidising motorist information revealed that there is no rationale to subsidise information as long as the government implements a proper fine congestion-pricing scheme. However, subsidising information may become an attractive (and necessary) policy option when a flat pricing scheme is adopted.

In the present paper we have assumed that the price of information is independent of the number of informed drivers. However, it may be more realistic to describe the costs of a motorist information system by a fixed cost component, reflecting the necessary equipment needed to provide information (for instance, road-side equipment or a central computer system), and a variable cost component, representing the costs of the in-vehicle unit (for instance, the on-board computer system). Such a cost structure would imply the existence of *economies of scale*. In future research, this deserves some attention, as it might influence the results.

Appendix A

A.1 Proof of Proposition

Proposition. *In a one-link network with endogenised demand for information, and assuming linear demand (D) and cost (C^0, C^1) functions, $C^0(N) \leq C^1(N)$, and $dC^0(N)/dN \leq dC^1(N)/dN$, the following relationships hold:*

- *(a) expected road usage is higher with information than without;*
- *(b) expected link travel costs are smaller with information than without;*
- *(c) expected welfare is higher with information than without.*

Proof. In the following we will assume that the linear demand curve D can be written as $D(N) = d - aN$, and the linear cost curves as $C^j(N) = k^j + b^j N$ for $j = 0, 1$.

First, we will derive the maximum possible price of information under which demand is non-negative. In state 0, this price ($\pi^0_{\max}$) can be found by equating N^0_{en} and N_n. Using expression (7) in Sect. 3.1 it follows that:

$$\pi^0_{\max} = (N^0_{ex} - N_n) \cdot (1-p) \cdot (a + b^0) \ . \tag{1}$$

A similar reasoning for state 1 yields:

$$\pi^1_{\max} = (N_n - N^1_{ex}) \cdot p \cdot (a + b^1) \ . \tag{2}$$

Using the explicit formulas for N_n, N^0_{ex}, it can easily be shown that $\pi^0_{\max} = \pi^1_{\max} (= \pi_{\max})$. Next, we will prove Proposition (a). First, using the equilibrium condition of model n (no information available), it follows that:

$$N_n = D^{-1}((1-p)\cdot C^0(N_n)+p\cdot C^1(N_n)) \ . \tag{3}$$

Then, using the equilibrium conditions of model *en* (endogenous demand for information) and the linearity of the demand function of road usage *D*, it can be shown that:

$$\begin{aligned} E(N_{en}) = (1-p)\cdot N^0_{en}+p\cdot N^1_{en} &= D^{-1}((1-p)\cdot C^0(E(N_{en}))+p\cdot C^1(E(N_{en})) \\ &\quad +(1-p)\cdot p\cdot(N^0_{en}-N^1_{en})\cdot(b^0-b^1)) \end{aligned} \tag{4}$$

Since $N^0_{en}-N^1_{en}$ is positive, and b^0-b^1 negative, it follows that:

$$\Delta = (1-p)\cdot p\cdot(N^0_{en}-N^1_{en})\cdot(b^0-b^1)<0 \ . \tag{5}$$

Writing $x=E(N_{en})$ and $y=N_n$, we have obtained the relationships:

$$\begin{aligned} x &= D^{-1}((1-p)\cdot C^0(x)+p\cdot C^1(x)+\Delta) \\ y &= D^{-1}((1-p)\cdot C^0(y)+p\cdot C^1(y)) \end{aligned} \tag{6}$$

Since Δ is smaller than zero, and D^{-1} is a decreasing function, it follows that x is greater than y. Hence, Proposition (a) holds true.

Next, since Δ is smaller than zero, Proposition (b) also holds true.

Finally, we will prove Proposition (c). We will do so by showing that welfare does positively depend on squared (expected) road usage. For the model without information (model n) this was proven in Emmerink et al. (1995b). Next, rewriting the expressions of the model in which information is endogenised (model en) we find:

$$C^0(N^0_{en}) = D(N^0_{en})-\frac{\pi}{1-p} \tag{7}$$

and

$$C^1(N^1_{en}) = D(N^1_{en})+\frac{\pi}{p} \tag{8}$$

Expected welfare is given by:

$$\begin{aligned} E(\text{Welfare}) &= (1-p)\cdot\left(\int_0^{N^0_{en}} D(x)\,dx-C^0(N^0_{en})\cdot N^0_{en}\right) \\ &\quad +p\cdot\left(\int_0^{N^1_{en}} D(x)\,dx-C^1(N^1_{en})\cdot N^1_{en}\right)-\pi\cdot(N^0_{en}-N^1_{en}) \\ &= (1-p)\cdot\left(\frac{1}{2}\cdot a\cdot(N^0_{en})^2+N^0_{en}\cdot\frac{\pi}{1-p}\right)+p\cdot\left(\frac{1}{2}\cdot a\cdot(N^1_{en})^2-N^1_{en}\cdot\frac{\pi}{p}\right) \\ &\quad -\pi\cdot(N^0_{en}-N^1_{en}) = \frac{1}{2}\cdot a\cdot E(N^2_{en}) \end{aligned} \tag{9}$$

where the last equality follows from the substitution of expressions (7) and (8) in expression (9). Proposition (c) now follows from applying Proposition (a). This completes the proof of Proposition.

A.2. Demand for information, expected link travel costs and social welfare

Under the assumptions of linear demand for travel ($D(N) = d - aN$) and linear link travel cost functions ($C^j(N) = k^j + b^j N$), it was shown in Sect. 3.1 that demand for information is linearly dependent on the private costs of being informed. Here we will prove that under these conditions expected link travel costs are also a linear function of demand for information. Expected link travel costs in the model with endogenous demand for information are given by:

$$
\begin{aligned}
(1-p)\cdot C^0(N^0_{en}) + p\cdot C^1(N^1_{en}) &= (1-p)\cdot C^0\left(N^0_{ex} - \frac{\pi}{(1-p)\cdot(a+b^0)}\right) \\
&\quad + p\cdot C^1\left(N^1_{ex} + \frac{\pi}{p\cdot(a+b^1)}\right) = \\
\text{Constant} + \pi\cdot\left(\frac{a\cdot(b^1-b^0)}{(a+b^0)\cdot(a+b^1)}\right), &\quad \text{where} \qquad (10) \\
\text{Constant} = (1-p)\cdot k^0 + p\cdot k^1 + (1-p)\cdot b^0\cdot N^0_{ex} + p\cdot b^1\cdot N^1_{ex} &
\end{aligned}
$$

Hence, the private costs of information π and the expected link travel costs are positively linearly dependent. Now, since demand for information and private costs of information π are negatively linearly dependent, it follows that the expected link travel costs are negatively linearly dependent on demand for information.

Finally, we will show that social welfare, as defined by total system benefits minus total system costs, is a convex quadratic increasing function of demand for information. We will do so by showing that the first derivative of welfare with respect to private costs of information π is negative, while the second derivative of welfare with respect to private costs of information π is positive, and the third derivative is equal to zero.

$$
\begin{aligned}
\frac{\partial\,\text{Welfare}}{\partial\pi} &= \frac{\partial\,\text{Welfare}}{\partial N^0_{en}}\cdot\frac{\partial N^0_{en}}{\partial\pi} + \frac{\partial\,\text{Welfare}}{\partial N^1_{en}}\cdot\frac{\partial N^1_{en}}{\partial\pi} \\
&= \frac{a\cdot(N^1_{en}\cdot(a+b^0) - N^0_{en}\cdot(a+b^1))}{(a+b^0)\cdot(a+b^1)} < 0 \qquad (11)
\end{aligned}
$$

This expression is negative because N^1_{en} is smaller than N^0_{en}, and b^0 is smaller than b^1.

$$
\frac{\partial^2\,\text{Welfare}}{\partial\pi^2} = a\cdot\left(\frac{1}{(1-p)\cdot(a+b^0)^2} + \frac{1}{p\cdot(a+b^1)^2}\right) > 0 \qquad (12)
$$

$$\frac{\partial^3 \text{Welfare}}{\partial \pi^3} = 0 \tag{13}$$

Hence, welfare is a convex quadratic decreasing function of the private costs of being informed π. Now, since the demand function for information is negatively linearly dependent on π, it follows that welfare is a convex quadratic increasing function of the number of informed drivers.

References

Arnott R, de Palma A, Lindsey R (1992) Route choice with heterogenus drivers and group-specific congestion costs. Reg Sci Urban Econ 22:71–102

Arnott R, de Palma A, Lindsey R (1993) A structural model of peak-period congestion: A traffic bottleneck with elastic demand. Am Econ Rev 83(1):161–179

Bonsall PW (1992) The influence of route guidance advice on route choice in urban networks. Transportation 19(1):1–23

Boyce DE (1988) Route guidance systems for improving urban travel and location choices. Transp Res 22A(4):275–281

El Sanhouri IM (1994) Evaluating the Joint Implementation of Congestion Pricing and Driver Information Systems. PhD thesis, Massachusetts Institute of Technology

Emmerink RHM, Verhoef ET, Nijkamp P, Rietveld P (1994a) Information provision in road transport with elastic demand: A welfare economic approach. Tinbergen Institute Discussion Paper, TI 94-144. Forthcoming in Journal of Transport Economics and Policy

Emmerink RHM, Nijkamp P, Rietveld P, Axhausen KW (1994b) The economics of motorist information systems revisited. Transp Rev 14(4):363–388

Emmerink RHM, Axhausen KW, Nijkamp P, Rietveld P (1995a) Effects of information in road transport networks with recurrent congestion. Transp 22(1):21–53

Emmerink RHM, Verhoef ET, Nijkamp P, Rietveld P (1995b) Information policy in road transport with elastic demand: Some welfare economic considerations. Tinbergen Institute Discussion Paper, TI 95-28

Emmerink RHM, Nijkamp P, Rietveld P, Van Ommeren JN (1996) Variable message signs and radio traffic information: An integrated empirical analysis of drivers' route choice behaviour. Transp Res 30A(2):135–153

Frey BS, Pommerehne WW (1993) On the fairness of pricing – An empirical survey among the general population. J Econ Behav Organ 20:295–307

Johansson B, Mattsson L-G (1995) Road Pricing: Theory, Empirical Assessment and Policy. Kluwer Academic Publishers, Dordrecht, The Netherlands

Kobayashi K (1994) Information, rational expectations and network equilibria – an analytical perspective for route guidance systems. Ann Reg Sci 28(4):369–393

Lotan T, Koutsopoulos HN (1993) Models for route choice behavior in the presence of information using concepts from fuzzy set theory and approximate reasoning. Transportation 20(2):129–155

Mahmassani HS, Jayakrishnan R (1991) System performance and user response under real-time information in a congested traffic corridor. Transp Res 25A(5):293–308

Verhoef ET, Emmerink RHM, Nijkamp P, Rietveld P (1994) Information provision, flat- and fine congestion tolling and the efficiency of road usage. Tinbergen Institute Discussion Paper, TI 94-157. Forthcoming in Regional Science and Urban Economics

Wardrop JG (1952) Some theoretical aspects of road traffic research. Proceedings of the Institute of Civil Engineers, 1 (Part II), pp 325–378

Yang H, Kitamura R, Jovanis PP, Vaughn KM, Abdel-Aty A (1993) Exploration of route choice behavior with advanced traveler information using neural network concepts. Transportation 20:199–223